equation." The author really wonders if Lam has read this paper thoroughly.

This paper also has indicated no intension to investigate the complicated processes between the striking particles and the satellite surface, but employs only the conditions of steady state of satellite potential and constancy of satellite mass. The diameter of the satellite has been considered to be rather large, of the order of meters. In general, such a satellite is constructed as a thin conducting shell. It is obvious that the electrical current normal to the satellite surface must be zero. Spitzer[1] also stated that "if the potential of the solid surface is allowed to float, no current must flow from the plasma to the surface." In Eq. (13), $n_{oe}e^{+g\psi_0/kT}$ and $(C_- + W\cos\theta)$ are the electron number density and their total mean speed in the direction normal to the satellite surface. The product of these two quantities is the contribution to electric current by electrons. In the same way, the left side of the equation is the contribution by ions. For zero current, Eq. (13) is established easily.

Lam's statement of "we see that Jen has a satellite made of a nonconductor" is certainly not true, since the paper has not used the term "nonconductor" at all, either explicitly or implicitly. Lastly, Lam's item 4 on Coulomb drag even contradicts his own item 1.

Reference

[1] Spitzer, L., Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), p. 17.

Comment on "General Instability and Optimum Design of Grid-Stiffened Spherical Domes"

Kenneth P. Buchert*
University of Missouri, Columbia, Mo.

GRID-STIFFENED spherical domes subject to external pressure, as suggested by Crawford and Schwartz,[1] have been designed and built for a number of years. Shells with base diameters of over 100 ft have operated successfully as roofs, parts of space simulators, and outer walls of cryogenic vessels.

Theoretical results and experimental data for general instability and for buckling between stiffeners have been published by the author.[2–4] The theoretical results[3] for a stiffened shell give the critical buckling pressure for general instability as

$$p_{cr} = 0.365E\,(t_s/R)^2\,[1 + (12I/t_s^3b_s)]^{1/2}\,[1 + (A/t_sb_s)]^{1/2}$$

when I is the effective moment of inertia of the stiffener, and A is the area of the stiffener.

The critical buckling pressure for local instability[3] is, approximately,

$$p_{cr} = 7.42Et_s^3/Rb_s^2$$

when the torsional stiffness of the stiffeners is relatively high. When the torsional stiffness is relatively low, the local buckling pressure can be calculated by using the method previously published.[4]

The results of tests[2] indicate that if local buckling occurs at a relatively low pressure, general instability soon follows, and the theoretical general stability pressure is very low. In addition, edge effects are important in the design of stiffened shells. A series of tests conducted at the University of Missouri indicate that general instability will occur at a very low load if poor edge conditions are present. In order to reach the theoretical value of the buckling pressure, the yield strain of the material must be high enough. Even though the strain in the shell and stiffeners just prior to buckling might be relatively low (considerably below the yield strain of the material), the strains during and after buckling are relatively high (exceeding the yield strain) in a practical shell.[5] This pseudoelastic effect has been demonstrated in tests performed at the University of Missouri.

The connections between the stiffeners and between the stiffeners and the shell are of considerable interest to the engineer. A limited number of tests have shown that the general instability critical pressure is only reduced about 10% if the connections between the stiffeners are eliminated and if the attachments between the stiffeners and shell are only 50% effective.

Received May 26, 1965.
* Associate Professor, Department of Civil Engineering.

References

[1] Crawford, R. F. and Schwartz, D. B., "General instability and optimum design of grid-stiffened spherical domes," AIAA J. **3**, 511–515 (1965).

[2] Buchert, K. P., "Stability of doubly curved stiffened shells," Ph.D. Dissertation, University of Missouri, Columbia, Mo. (January 1964).

[3] Buchert, K. P., "Stiffened thin shell domes," AISC, **7**, 78–82 (1964).

[4] Buchert, K. P., "Zur Stabilität grosser, doppelt gekrümmter und versteifter Schalen," Stahlbau **2**, 55–62 (1965).

[5] Buchert, K. P., "Buckling of doubly curved orthotropic shells," Engineering Experiment Station, University of Missouri, Columbia, Mo. (November 1965).

Reply by Authors to K. P. Buchert

Robert F. Crawford* and David B. Schwartz†
Martin Company, Baltimore, Md.

STIFFENED shells of various types have been designed, built, and used in flight vehicles as well as in civil structures for a number of years; however, there still exists a potential for further reducing their weight and improving the accuracy of their analysis. Substantial improvements in vehicle performance that can be gained from such weight reductions provide the impetus for accurately defining the potential minimum weight and associated design details for the many classes of stiffened shells used in their construction.

The general instability formula presented in the previous comment is an approximation to the critical pressure for symmetric buckling with a reduction factor derived from a Karman- and Tsien-type buckling analysis. Perhaps Professor Buchert's procedure is justifiable for monocoque or sandwich shells whose symmetric and asymmetric modes of buckling have equal critical pressures; however, the subject paper shows [Eqs. (13) and (14)] that the critical pressure for asymmetric buckling is lower by a factor of $[(1 + D_3/D)/(1 + E/G_3)]^{1/2}$ for equal stiffeners in the orthogonal directions when $D_3/D < E/G_3$, as it is for the square-grid-stiffening case. In terms of parameters similar to those used by Professor Buchert previously, the small-deflection theory formula

Received September 23, 1965.
* Senior Staff Engineer, Solid Mechanics Section. Member AIAA.
† Senior Engineering Specialist, Solid Mechanics Section.

for critical pressure is

$$p_{cr} = \frac{4E}{[12(1-\mu^2)]^{1/2}}\left(\frac{t_s}{R}\right)^2\left\{\frac{\left[\frac{1+(A/bt_s)}{1+(1-\mu^2)\,A/bt_s}+\frac{A}{bt_s}\right]\left[1+\frac{12(1-\mu^2)I}{bt_s^3}+\frac{12(1-\mu^2)(d/t_s)^2}{1-\mu^2+(bt_s/A)}\right]}{2+(1+\mu)\,A/bt_s}\right\}^{1/2}$$

where I is the actual moment of inertia of the stiffener, and d is the distance from the skin midplane to the stiffener centroid. Equation (19) of the subject paper was used for E/G_3, and D_3/D was assumed negligible relative to unity.

The important differences between this formula and Professor Buchert's are the added term in the denominator necessary to lower the critical pressure to that for asymmetric buckling, and the larger numerical coefficient $4[12(1-\mu^2)]^{-1/2}$, which is the result of using classical theory. Perhaps the experimental results referred to in the previous comment should be correlated to this formula to provide a reduction factor and, if necessary, referred to a more appropriate formula. That is, a paper by Crawford[1] contains a further linear-theory correction to the previous formula that accounts for the asymmetric section geometry of this class of stiffening. It would be even more appropriate to use those results to establish a proper correlation factor between linear theory and experiment. The correction is made by simply adding to the previous equation the term

$$\frac{2EAd}{R^2b}\left\{\frac{1-(2\mu)/[1+(1-\mu^2)\,A/bt_s]}{2+(1+\mu)\,A/bt_s}\right\}$$

for stiffening on the convex surface. The term is subtracted for stiffening on the concave surface.

Equation (10) of the subject paper reduces to the following formula for critical pressure for local instability when the stiffeners provide nodes but no torsional restraint:

$$p_{cr} = \frac{3.62\,Et_s^3}{Rb_s^2}\left[1+\frac{3\,(1-\mu^2)}{\pi^4}\left(\frac{b}{R}\right)^4\left(\frac{R}{t_s}\right)^2\right]$$

For geometric proportions in the vicinity of their optimum value, the second term in this formula for local instability is less than 1% when $p/E < 10$. Equations (16) and (30), which are equal to the previous equation when its second term is neglected, are therefore appropriate as recommended in the paper when the stiffener's torsional stiffness is neglected for conservatism.

References

[1] Crawford, R. F., "Effects of asymmetric stiffening on buckling of shells," AIAA Preprint 65-371 (July 26–29, 1965).

Comments on "In-Plane Vibration of Spinning Disks"

J. G. Simmonds*
Harvard University, Cambridge, Mass.

THERE are three aspects of the subject note[1] to be commented upon. First, since none of the four references cited by Huston deal with the in-plane vibrations of a spinning disk, the reader may have been left with the impression that no previous work had been done on this problem. The fact is that at least six earlier papers, Refs. 2–7, have dealt with the more complicated problem of the symmetric (and, in some cases, unsymmetric) in-plane vibrations of a spinning elastic disk attached to a hub of nonzero radius which includes, as a limiting case, the freely spinning disk.

Second, Eqs. (22) and (23) of Ref. 1 are of questionable physical significance, since the relative order of magnitude of the coriolis effects $\zeta R^2\Omega^2/E$ is of the same order of magnitude as the static strain produced in the disk by the centrifugal loading, and Huston's governing linear equations (1) and (2) already neglect terms of this order.

Third, in this writer's opinion, a satisfactory discussion of the effects of rotation on the frequencies of vibration of a spinning disk is yet to be given. For example, depending on whether the seemingly negligible terms $-\Omega^2u$ and $-\Omega^2v$ are added or not to the right-hand sides of Eqs. (1) and (2) of Ref. 1, one finds that, as the rotational frequency Ω increases, the lowest torsional mode of a disk attached to a central hub becomes unstable[4,7] or remains stable,[2,3,6] respectively. Thus, were Huston's equations to be applied to this problem, his argument for neglecting these terms as simply being small[8] would have to be modified. It appears that any complete treatment of the problem should include the effects of the initial, static stresses on the vibration. Some discussion on this point may be found in Ref. 6.

References

[1] Huston, R. L., "In-plane vibration of spinning disks," AIAA J. **3**, 1519–1520 (1965).
[2] Grammel, R., "Drillungs-und Dehungsschwingungen unlaufender Schieben," Ing-Arch. **6**, 256–265 (1935).
[3] Biezeno, C. B. and Grammel, R., *Engineering Eynamics: Steam Turbines*, translated by E. F. Winter and H. A. Havemann (Blackie and Sons, Ltd., London, 1954), Vol. III.
[4] Yamada, K., "Vibration of turbine disc in its plane," *Proceedings of the 2nd Japan National Congress Applied Mechanics* (National Committee on Theoretical and Applied Mechanics, Science Council of Japan, Tokyo, 1953), pp. 343–347.
[5] Singh, B. R. and Nandeeswaraiya, N. S., "Vibration analysis of turbine disc in its plane," J. Sci. Eng. Res. (India) **1**, 157–160 (1957).
[6] Simmonds, J. G., "The in-plane vibration of a flat spinning disk," NASA TN D-521 (December 1962).
[7] Bhuta, P. G. and Jones, J. P., "Symmetric planar vibrations of a rotating disk," J. Acoust. Soc. Am. **35**, 982–989 (1963).
[8] Huston, R. L., "Wave propagation in rotating elastic media," AIAA J. **2**, 575–576 (1964).

Received September 7, 1965.
* Research Fellow in Structural Mechanics.

Reply by Author to J. G. Simmonds

Ronald L. Huston*
University of Cincinnati, Cincinnati, Ohio

PERHAPS it is convenient to reply to these statements in order. First, I extend my apologies to J. G. Simmonds and any others I may have overlooked in the references. I chose primarily those works that I needed for the development of the subject note,[1] the purpose being to investigate the effect of the Coriolis acceleration. In much of the work done by others,[2,3] this effect apparently is discarded as being negligible.

Second, a mathematical model can only be expected to predict physical phenomena in view of the assumptions used in

Received September 27, 1965.
* Assistant Professor, Department of Mathematics and Mechanics.